

Different problems and parts may not have equal value.

Make sure to differentiate between transitive set TS and transitive relation TR. The word transitive by itself is not clear (so that full credit cannot be given). To receive full credit, make sure to appropriately set up and end all proofs (especially in problems 3, 4, and 5).

1. (a) Provide “elementary” maps that verifying that $|\omega| = |\mathbb{Q}^{<\omega}|$. Indicate which maps are 1-1 and/or onto, and indicate what bijection verifies the actual equality listed. (Do not forget to take care of 0, and when appropriate, 1, as they are elements of ω .)

- (b) Prove $|A| \neq |\wp(A)|$ for any set A .

To receive full credit, show all steps and appropriately set up and end your proof.

2. Suppose $a < b$ and A is a nonempty finite set disjoint from (a, b) .

Use “elementary maps” to show $|(a, b) \cup A| = |\mathbb{R}|$.

Indicate which maps are 1-1 and/or onto, and indicate what bijection verifies the actual equality listed.

3. (a) Prove that “ON satisfies the WOP.”

- (b) Prove that “ON satisfies trichotomy.” Hint: You will likely need to first state and prove a Lemma.

4. Assume that κ is an infinite cardinal such that there exist $\lambda < \kappa$ and $\langle S(\xi) \mid \xi < \lambda \rangle$ such that $|S(\xi)| < \kappa$ and $\kappa = \bigcup_{\xi < \lambda} S(\xi)$. Prove that κ is singular, i.e. show that $cf(\kappa) < \kappa$.

5. Prove that if κ is an infinite cardinal, then $\kappa < \kappa^{cf(\kappa)}$.

$\kappa^{cf(\kappa)}$ involves cardinal exponentiation and is therefore a cardinal. It is the cardinal $|\kappa^{cf(\kappa)}|$ when we instead interpret $\kappa^{cf(\kappa)}$ to be the set of all functions from $cf(\kappa)$ into κ .