

MAT 701, Fall 2009, Test 1, Date_____ Name_____

Different problems and parts do not have equal value. Use Cantor Diagonalization in problems 2 and 4(a). To receive full credit, show all steps and appropriately set up and end your proofs (especially problems 2 and 4). Do problems 1-4 (not 5 if accidentally included).

1. Provide “elementary” maps that verify the equalities below. Indicate which maps are 1-1 and/or onto, and indicate what bijection verifies the actual equality listed. (Do not forget to take care of 0, and when appropriate, 1, as they are elements of ω .)

(a) $|\omega| = |\mathbb{Q}|$

(b) $|\omega| = |\mathbb{Q}^3|$

(c) $|\omega| = |\mathbb{Q}^{<\omega}|$

2. Prove that $\mathbb{Q}^{\mathbb{N}}$ is uncountable.

To receive full credit, show all steps and appropriately set up and end your proof.

3. Provide “elementary” maps that verify the set Alg of algebraic numbers is denumerable. Indicate which maps are 1-1 and/or onto, and indicate what bijection verifies that Alg is denumerable.

4. (a) Prove $|A| \neq |\mathcal{P}(A)|$ for any set A .

To receive full credit, show all steps and appropriately set up and end your proof.

(b) Use part (a) to prove $|A| < |\mathcal{P}(A)|$ for any set A .

To receive full credit, show all steps and appropriately set up and end your proof.

(c) Prove that for any set U there exists a set $E \notin U$.