

Different problems and parts may not have equal value.

Make sure to differentiate between transitive set TS and transitive relation TR. The word transitive by itself is not clear (so that full credit cannot be given). To receive full credit, make sure to appropriately set up and end all proofs (especially in problems 3, 4, 5, and 6d).

1. (a) Outline the proof of the Bernstein Schroeder Theorem: If $|A| \leq |B|$ and $|B| \leq |A|$, then $|A| = |B|$.
 (b) If $f : A \xrightarrow{1-1} B$ and $g : B \xrightarrow{1-1} A$, then $B \setminus \text{Im } f \subseteq \underline{\hspace{2cm}}$ and $A \setminus \text{Im } g \subseteq \underline{\hspace{2cm}}$.

2. Suppose $a < b$ and B is a denumerable set (same size as $\mathbb{N}$) that is disjoint from (a, b) .

Use “elementary maps” to show $|(a, b) \cup B| = |\mathbb{R}|$.

Indicate which maps are 1-1 and/or onto, and indicate which bijection verifies $|(a, b) \cup B| = |\mathbb{R}|$.

3. Use Cantor Diagonalization to prove that there is a transcendental number in the interval $(5.9999, 6)$.

Recall from Test 1 that the set Alg of algebraic numbers is denumerable.

Note that a transcendental number is a number that is not algebraic.

To receive full credit, show all steps and appropriately set up and end your proof.

4. Prove that “ON satisfies the WOP.”

To receive full credit, show all steps and appropriately set up and end your proof.

5. Prove that “ON satisfies trichotomy.” Hint: You may need to first state and prove a Lemma.

To receive full credit, show all steps and appropriately set up and end your proof.

6.

- (a) (From $\in$ is a transitive relation on ON.)

Suppose $x \in y \in z \in \text{ON}$. $x \in z$ follows from what property of z or $(z, \in)$: _____

- (b) (From ON is a transitive class, i.e. satisfies the transitive set property TS.)

Suppose $y \in z \in \text{ON}$. “ y being a transitive set” follows from what properties of z or $(z, \in)$: _____

- (c) (From ON is a transitive class, i.e. satisfies the transitive set property TS.)

Suppose $x \in y \in \text{ON}$. That $(x, \in)$ satisfies trichotomy follows from what properties of $(y, \in)$: _____

- (d) Prove that ON is not a set.

To receive full credit, show all steps and appropriately set up and end your proof.