

Math 751 Fall 2010 Final. Make sure to appropriately set up and end each proof.

1. Let $A \subseteq X^\omega$ be open. Prove that if the empty sequence $\langle \rangle \notin \text{Ord}_A$, then player II has a winning quasi-strategy for the game A . Make sure to appropriately set up and end your proof.
2. Prove the following Theorem of Martin: If there exists a measurable cardinal, then all boldface Π_1^1 games (with moves from ω) are determined.

You may cite Quiz 5 for Kleene's Theorem, for the definition and winning conditions of the auxiliary game G^* , and that G^* has a winning strategy s^* . You may cite the homework for the fact that if s^* is a winning strategy for player I^* , then player I has a winning strategy in the original game. In the appropriate case, make sure to:

- prove the needed Ramsey-type property and
- define a strategy s for player II in the original game.

You may cite Quiz 6 for the fact that such s is in fact a winning strategy for player II in the original game. Then appropriately finish your proof.

3. Prove that for every ordinal α , $L_\alpha \cap \text{ON} = \alpha$.

4. Prove the Main Lemma for our Reflection Theorem about L .

For every formula $\varphi = \varphi(x_0, x_1, x_2, \dots, x_n)$ and every ordinal α_0 , there exists an ordinal $\alpha > \alpha_0$ such that for all $u_1, u_2, u_3, \dots, u_n \in L_\alpha$:

$$\text{If } \exists x \in L \ \varphi(x, u_1, u_2, \dots, u_n), \text{ then } \exists x \in L_\alpha \ \varphi(x, u_1, u_2, \dots, u_n).$$