

Quiz 1 MAT 751: Suppose that player I does not have a winning strategy for a game $A \subseteq X^\omega$. Prove that player II has a quasi-strategy s such that for every $n \in \omega$, if $f \upharpoonright 2n = (f(0), f(1), f(2), \dots, f(2n-1))$ is according to s , then player I does not have a winning strategy for the game A at $f \upharpoonright 2n$, i.e. for the game $A_{f \upharpoonright 2n}$. Recall that s is called the non-losing quasi-strategy for player II, even though there may be plays according to s which are a loss for player II. Make sure to appropriately set up and end your proof.

Quiz 2 MAT 751: Prove that if player I does not have a winning strategy for an open game $A \subseteq X^\omega$, then the non-losing quasi-strategy for player II is a winning strategy. Make sure to appropriately set up and end your proof.

Quiz 3 MAT 751: Let $A \subseteq X^\omega$ be open.

- Define $Ord_A(\alpha) \subseteq X^{<\omega}$ for each ordinal α so that $Ord_A(\alpha) \subseteq Ord_A(\beta)$ if $\alpha < \beta$. Also define Ord_A .
- Prove that if the empty sequence $\langle \rangle \in Ord_A$, then player I has a winning quasi-strategy for the game A . Make sure to appropriately set up and end your proof.

Quiz 4 MAT 751: Let $A \subseteq X^\omega$ be open. Prove that if the empty sequence $\langle \rangle \notin Ord_A$, then player II has a winning quasi-strategy for the game A . Make sure to appropriately set up and end your proof.

Quiz 5 MAT 751: Recall the following Theorem of Martin: If there exists a measurable cardinal, then all boldface Π_1^1 games (with moves from ω) are determined.

- State the Kleene Theorem that will be used to define the auxiliary game used in the proof of Martin's Theorem.
- Define the auxiliary game G^* and indicate why G^* has a winning strategy s^* . Don't forget the winning conditions are part of the definition of G^* .

Quiz 6 MAT 751: Now finally suppose a measurable cardinal κ exists with a normal measure D .

Prove that if s^* is a winning strategy for player II^* , then there exists $H \in D$ such that for all $n < \omega$, $\forall \xi_0, \xi_1, \xi_2, \dots, \xi_n \in H \ \forall \lambda_0, \lambda_1, \lambda_2, \dots, \lambda_n \in H \ \forall x(0), x(1), x(2), \dots, x(2n) \in \omega$

if neither position $(x(0), \xi_0; x(2), \xi_1; x(4), \xi_2; \dots; x(2n), \xi_n)$ nor $(x(0), \lambda_0; x(2), \lambda_1; x(4), \lambda_2; \dots; x(2n), \lambda_n)$ is a loss for player I^*

(i.e. if the two maps $(n+1, <_{\bar{x}(n+1)}) \rightarrow \kappa$ defined by $i \mapsto \xi_i$ and $i \mapsto \lambda_i$ are order preserving), then

$$s^*(x(0), \xi_0; x(2), \xi_1; x(4), \xi_2; \dots; x(2n), \xi_n) = s^*(x(0), \lambda_0; x(2), \lambda_1; x(4), \lambda_2; \dots; x(2n), \lambda_n).$$

Quiz 7 MAT 751: Assuming κ is a measurable cardinal with a normal measure D and s^* is a winning strategy for player II^* , use Quiz 6 to define a strategy s for player II in the original game and prove s is in fact a winning strategy in the original game.