

MAT 701, Fall 2011, Final, Date_____ **Name**_____

Makes sure to set up and end all proofs.

2. Suppose $E \subseteq \mathbb{R}$ is infinite and there exists $g : \omega \xrightarrow{\text{onto}} E$. Provide “elementary” maps that verify the set $\text{Alg}(E)$ of algebraic numbers over E is denumerable (again without using Bernstein Schroeder).

Indicate which maps are 1-1 and/or onto, and indicate what bijection verifies that Alg is denumerable.

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4. Prove that for every (nonempty) perfect set $P \subseteq \mathbb{R}$ there exists an injection $\Psi : \{0,1\}^{\omega} \xrightarrow{1-1} P$.

5. (a) Outline the proof of the Bernstein Schroeder Theorem: If $|A| \leq |B|$ and $|B| \leq |A|$, then $|A| = |B|$.

(b) If $f : A \xrightarrow{1-1} B$ and $g : B \xrightarrow{1-1} A$, then $B \setminus \text{Im } f \subseteq ______$ and $A \setminus \text{Im } g \subseteq ______$.

6. Let $F \subseteq \mathbb{R}$ be an uncountable closed set and assume $P \subseteq F$ is perfect (and nonempty). Use the Bernstein Schroeder Theorem and problems from Test 1 to show that $|F| = |\mathbb{R}| = |P| = |\wp(\omega)| = |\{0,1\}^{\omega}|$.

13. Prove the Perfect Set Theorem: for any uncountable closed set $F \subseteq \mathbb{R}$ there exists a perfect set P and a countable set C such that $F = P \dot{\cup} C$.

16. Assume $\alpha > 0$ is a limit ordinal and $\beta : \gamma \xrightarrow[\text{nondecreasing}]{\text{unbounded}} \alpha$. Define the strictly increasing map that “shows/witnesses” $cf(\gamma) \leq cf(\alpha)$.