

In the problems below, if you use the “skipping” function, make sure to provide its complete definition. Also in the problems below, do not forget to take care of 0, and when appropriate, 1, as they are elements of ω .

Quiz 1 consists of problems 1-3, Quiz 2 is just problem 4, etc. You can take these quizzes on different days and/or times, since I have announced what is on these quizzes.

Quiz 1:

1. Fix a natural number $N \geq 1$. First define a map f from ω onto ω^N . If f is also 1-1, then indicate so. Otherwise, use f to define a map from ω 1-1 and onto ω^N .
2. First define a map g from ω onto $\mathbb{Q}$. If g is also 1-1, then indicate so. Otherwise, use g to define a map from ω 1-1 and onto $\mathbb{Q}$.
3. First define a map h from ω onto $\mathbb{Q}^{<\omega}$. If h is also 1-1, then indicate so. Otherwise, use h to define a map from ω 1-1 and onto $\mathbb{Q}^{<\omega}$. Feel free to use any map(s) from problem 2.

Quiz 2:

4. Suppose A is an infinite subset of $\mathbb{R}$ and $g : \omega \rightarrow A$ is an onto map. First use g to define an onto map $h : \omega \rightarrow \text{Alg}(A)$. Then use h to define a 1-1, onto map from ω onto $\text{Alg}(A)$.

Quiz 3:

5. Suppose $a < b$, A is a nonempty finite set, B is a denumerable set (same size as ω), and both A and B are disjoint from (a, b) . Use “elementary maps” to show: (i) $|(a, b) \cup A| = |\mathbb{R}|$ and (ii) $|(a, b) \cup B| = |\mathbb{R}|$. Indicate which maps are 1-1 and/or onto, and indicate what bijection verifies the actual equality listed.

Quiz 4:

6. (a) Outline the proof of the Bernstein Schroeder Theorem: If $|A| \leq |B|$ and $|B| \leq |A|$, then $|A| = |B|$.
- (b) If $f : A \xrightarrow{1-1} B$ and $g : B \xrightarrow{1-1} A$, then $B \setminus \text{Im } f \subseteq \underline{\hspace{2cm}}$ and $A \setminus \text{Im } g \subseteq \underline{\hspace{2cm}}$.

Quiz 5:

7. Prove that for every (nonempty) perfect set $P \subseteq \mathbb{R}$ there exists an injection $\Psi : \{0, 1\}^{\omega} \xrightarrow{1-1} P$.

Quiz 6:

8. Prove that there exists a universal Σ_1^0 set. Once you define the needed set, feel free to indicate that it is obvious that the “slices” are exactly all the open sets in the space ω^{ω} and then finish by proving that the set you defined is in fact open in $\omega^{\omega} \times \omega^{\omega}$.

Quiz 7:

9. Assume that there exists a universal Σ_{α}^0 set U . Use U to define a Π_{α}^0 set D that is not Σ_{α}^0 . Feel free to mention that $D \in \Pi_{\alpha}^0 | \omega^{\omega}$ whereas prove D is not Σ_{α}^0 .