

MAT 701, Fall 2011, Test 1, Date_____ Name_____

Different problems and parts do not have equal value. In particular, problem 4 has much more value than any of the other problems. To receive full credit in problem 4, show all steps and appropriately set up and end your proof.

1. Provide “elementary” maps that verify the equalities below (without using Bernstein Schroeder). Indicate which maps are 1-1 and/or onto, and indicate what bijection verifies the actual equality listed. (Do not forget to take care of 0, and when appropriate, 1, as they are elements of ω .)

(a) $|\omega| = |\mathbb{Q}|$

(b) $|\omega| = |\mathbb{Q}^3|$

(c) $|\omega| = |\mathbb{Q}^{<\omega}|$

2. Suppose $E \subseteq \mathbb{R}$ is infinite and there exists $g : \omega \xrightarrow{\text{onto}} E$. Provide “elementary” maps that verify the set $\text{Alg}(E)$ of algebraic numbers over E is denumerable (again without using Bernstein Schroeder).

Indicate which maps are 1-1 and/or onto, and indicate what bijection verifies that Alg is denumerable.

3. Suppose $a < b$, A is a nonempty finite set, B is a denumerable set (same size as ω), and both A and B are disjoint from (a, b) . Use “elementary maps” to show (again without using Bernstein Schroeder):

(i) $|(a, b) \cup A| = |\mathbb{R}|$ and (ii) $|(a, b) \cup B| = |\mathbb{R}|$.

Indicate which maps are 1-1 and/or onto, and indicate what bijection verifies the actual equality listed.

4. Prove that for every (nonempty) perfect set $P \subseteq \mathbb{R}$ there exists an injection $\Psi : \{0, 1\}^{\omega} \xrightarrow{1-1} P$.