

Name _____ Email address _____
MAT 701, Fall 2011, Test 2, Date: _____

Different problems and parts may not have equal value. Make sure to differentiate between transitive set TS and transitive relation TO. The word transitive by itself is not clear (so that full credit cannot be given). To receive full credit, make sure to appropriately set up and end all proofs.

1. (a) Outline the proof of the Bernstein Schroeder Theorem: If $|A| \leq |B|$ and $|B| \leq |A|$, then $|A| = |B|$.
 (b) If $f : A \xrightarrow{1-1} B$ and $g : B \xrightarrow{1-1} A$, then $B \setminus \text{Im } f \subseteq$ _____ and $A \setminus \text{Im } g \subseteq$ _____.

2. Let $F \subseteq \mathbb{R}$ be an uncountable closed set and assume $P \subseteq F$ is perfect (and nonempty). Use the Bernstein Schroeder Theorem and problems from Test 1 to show that : $|F| = |\mathbb{R}| = |P| = |\wp(\omega)| = |\{0,1\}^\omega|$.

3. (a) Prove $|A| \neq |\wp(A)|$ for any set A .
 To receive full credit, show all steps and appropriately set up and end your proof.

 (b) Use part (a) to prove $|A| < |\wp(A)|$ for any set A .
 To receive full credit, show all steps and appropriately set up and end your proof.

 (c) Prove that for any set U there exists a set $E \notin U$.

4. Prove that “ ON satisfies the WOP.”
 To receive full credit, show all steps and appropriately set up and end your proof.

5. Prove that “ ON satisfies trichotomy.” Hint: You may need to first state and prove a Lemma.
 To receive full credit, show all steps and appropriately set up and end your proof.

6.
 - (a) (From $\in$ is a transitive relation on ON .) Suppose $x \in y \in z \in ON$. $x \in z$ follows from what property of $(z, \in)$ out of $IR(z)$, $TO(z)$, $TRICH(z)$, $TS(z)$, $WO(z)$: _____

 - (b) (From ON is a transitive class, i.e. satisfies the transitive “set” property TS.)
 Suppose $y \in z \in ON$. “ y being a transitive set” $TS(y)$ follows from what two properties of $(z, \in)$ out of $IR(z)$, $TO(z)$, $TRICH(z)$, $TS(z)$, $WO(z)$: _____

 - (c) (From ON is a transitive class, i.e. satisfies the transitive set property TS.)
 Suppose $y \in z \in ON$. That $(y, \in)$ satisfies $LO(y)$ follows from what two properties of $(z, \in)$ out of $LO(z)$, $IR(z)$, $TO(z)$, $TRICH(z)$, $TS(z)$, $WO(z)$: _____

 - (d) Prove that ON is not a set.
 To receive full credit, show all steps and appropriately set up and end your proof.