

Name _____ Email address _____
MAT 701, Fall 2011, Test 3, Date: _____

Different problems may not have equal value.

To receive full credit, make sure to appropriately set up and end all proofs.

1. Prove that there exists a universal Σ_1^0 set. Once you define the needed set, feel free to indicate it is obvious that the “slices” are exactly all the open sets in the space ω^ω and then finish by proving that the set you defined is in fact open in $\omega^\omega \times \omega^\omega$.
2. Assume that there exists a universal Σ_α^0 set U . Use U to define a set $D \in \Pi_\alpha^0 \mid \omega^\omega$ that is not Σ_α^0 . Feel free to mention that $D \in \Pi_\alpha^0 \mid \omega^\omega$ is routine to show by considering the appropriate continuous map, whereas prove D is not Σ_α^0 .
3. Prove the Perfect Set Theorem: for any uncountable closed set $F \subseteq \mathbb{R}$ there exists a perfect set P and a countable set C such that $F = P \dot{\cup} C$.
To receive full credit, show all steps and appropriately set up and end your proof.
4. Prove that P and C in the previous problem (corresponding to a fixed $F \subseteq \mathbb{R}$) are unique.
To receive full credit, show all steps and appropriately set up and end your proof.
5. Prove that if α is an infinite limit ordinal, then $\text{cf}(\text{cf}(\alpha)) = \text{cf}(\alpha)$.
Feel free to assume that $\text{cf}(\alpha)$ is an infinite limit ordinal.