

Quizzes

To the proctor: Please note that Quiz 5 has part of the solution to quiz 4, so that quiz 5 cannot be handed out with quiz 4.

Quiz 1. Let κ be a regular uncountable cardinal. Prove that the intersection of two club subsets of κ is also club.

Quiz 2. Let κ be a regular uncountable cardinal, $\lambda < \kappa$, and $\langle C_\alpha \mid \alpha < \lambda \rangle$ be a sequence of club subsets of κ . Show $\bigcap_{\alpha < \lambda} C_\alpha$ is club.

Quiz 3. Let κ be a regular, uncountable cardinal and $\langle C_\alpha \mid \alpha < \kappa \rangle$ be a sequence of club subsets of κ . Show $\bigtriangleup_{\alpha < \kappa} C_\alpha$ is also club.

Quiz π : Prove Fodor's Theorem: Let κ be a regular uncountable cardinal. If f is a regressive function on a stationary set $S \subseteq \kappa$, then there exist stationary $T \subseteq S$ such that f restricted to T is constant.

Quiz 4. Recall the following theorem: Let κ be a regular, uncountable cardinal. Every stationary subset of E_ω^κ is the union of κ pairwise disjoint stationary sets. State and prove the main lemma to this theorem.

Quiz 5. Let κ be a regular, uncountable cardinal and let W be a stationary subset of E_ω^κ . For each $\alpha \in W$, let $f^\alpha : \omega \xrightarrow[\text{unbounded}]{\text{increasing}} \alpha$. Assume the Main Lemma: there exists $i < \omega$ such that for all $\eta < \kappa$, we have $F_i^\eta =_{df} \{\alpha \in W \mid f^\alpha(i) \geq \eta\}$ is stationary. Using the Main Lemma, prove that W is the union of κ pairwise disjoint stationary sets.

Quiz 6. Let κ be a regular, uncountable cardinal, let A be a stationary subset of κ , and let $A^L = \{\alpha \in A \mid \alpha \text{ is a limit ordinal (in particular infinite)}\}$. Recall that it is easy to show that one of two sets $A^S = \{\alpha \in A \mid \text{cf}(\alpha) < \alpha\}$ and $A^R = \{\alpha \in A \mid \alpha \text{ is a regular uncountable cardinal}\}$ is stationary. Assume A^R is stationary, in which case one can show $W = \{\alpha \in A^R \mid \alpha \cap A^R \text{ is not a stationary subset of } \alpha\}$ is a stationary subset of κ so that $\forall \alpha \in W \exists K^\alpha (K^\alpha \text{ is club in } \alpha \text{ and disjoint from } \alpha \cap A^R)$. For each $\alpha \in W$, let $f^\alpha : \alpha \xrightarrow[\text{continuous}]{\text{unbounded}} K^\alpha$ enumerate K^α . Prove that following Main Lemma: there exists $i < \kappa$ such that for all $\eta < \kappa$, we have $F_i^\eta =_{df} \{\alpha \in W \mid \alpha > i \text{ and } f^\alpha(i) \geq \eta\}$ is stationary.

Quiz 7. Prove Ramsey's Theorem that $\aleph_0 \rightarrow (\aleph_0)_k^n$ for all $n < \omega$ and all $k < \omega$.