

Spring 2010 Math 702 TEST 3

As always, make sure to cite any hypotheses, any previous results or problems used, etc. at the appropriate places in you proofs. Also, make sure to appropriately start and end your proofs. Problems do not have the same value in computing your test grade; in particular, problem 1 is clearly worth less.

1. Let κ be a regular uncountable cardinal and F be a normal filter that contains (as elements) all final segments (α, κ) . From this, one can show F contains all club subsets of κ . Let C be an arbitrary club subset of κ and let $\varphi: \kappa \xrightarrow[\text{continuous}]{\text{unbounded}} C$ enumerate C . Using φ , write down the element of F which is an intersection of diagonal intersections of final segments and thus is an element of F .

(From #1, Lemma 10.19 easily follows: if D is a normal measure on a measurable cardinal κ , then every element of D is stationary. Recall that you will likely use #1 and this Lemma in problem #4.)

2. Prove that every measurable cardinal is inaccessible (regular, strong limit, and uncountable). Recall that uncountable is part of the definition of a measurable cardinal.

3. Prove that every measurable cardinal carries a normal measure.
Recall you may choose to indicate that it is routine to show the order which you define on the equivalence classes is a linear order and that you may cite what is needed show this ordering is a well-ordering. You may also cite that it is routine to use the properties of the original nonprincipal κ -complete ultrafilter and the function that you will select to show that the normal measure that you define is in fact also a nonprincipal κ -complete ultrafilter, thus leaving to show that it is normal.

4. Prove that if D is a normal measure on a measurable cardinal κ , then $I^\kappa = \{\alpha < \kappa \mid \alpha \text{ is inaccessible}\} \in D$ and I^κ is stationary.

5. Prove the induction step of the following theorem: If D is a normal measure on a measurable cardinal κ , then for every $\varphi: [\kappa]^{<\omega} \rightarrow I$ such that $|I| < \kappa$ there exists $H \in D$ which is homogeneous for φ .