

HW 1 MAT 702:

Prove $\forall \alpha \in ON \neg \exists \beta \in ON (\alpha < \beta < S(\alpha))$ where $S(\alpha) \stackrel{df}{=} \alpha \cup \{\alpha\}$,

i.e., prove $\forall \alpha \in ON \forall \beta \in ON (\beta \leq \alpha \text{ or } S(\alpha) \leq \beta)$.

HW 2 MAT 702:

Assume $\alpha \neq 0$ is a limit ordinal.

(a) Prove that if $\exists f : \beta \xrightarrow{\text{unbounded}} \alpha$, then $\exists \delta \leq \beta (\exists S_f = g : \delta \xrightarrow[\text{strictly increasing}]{\text{unbounded}} \alpha)$.

(b) Define $\overset{\circ}{cf}(\alpha) = \mu \delta (\exists f : \delta \xrightarrow{\text{unbounded}} \alpha)$. Show $cf(\alpha) = \overset{\circ}{cf}(\alpha)$.

HW 3 MAT 702:

(1) Show that the following is false. "Composition of unbounded maps is unbounded map".

(2) "Composition of unbounded maps is unbounded map" if the appropriate map is nondecreasing.

HW 4 MAT 702:

(a) Show that if $\alpha, \beta \in ON$ and $\exists g : \alpha \xrightarrow{\text{order preserving}} \beta$, then $\alpha \leq \beta$.

(b) Show that above is false under the assumption that g is only 1-1.

HW 5 MAT 702:

There is no homework 5.

HW 6 MAT 702:

Assume κ is a regular uncountable cardinal.

(1) Every stationary subset W of $E_\lambda^\kappa = \{\alpha < \kappa \mid cf(\alpha) = \lambda\}$ for regular $\lambda < \kappa$ is the pairwise disjoint union of κ -many stationary subsets of κ .

(2) Every stationary subset of $\{\alpha < \kappa \mid cf(\alpha) = \alpha\}$ is the pairwise disjoint union of κ -many stationary subsets of κ .

Assume (1) and (2). Prove that every stationary subset S of κ is the pairwise disjoint union of κ -many stationary sets.

HW 7 MAT 702:

Let C be a club subset of a regular cardinal $\kappa \geq \omega_1$.

(a) Let $C' = \left\{ \alpha < \kappa \mid \underbrace{\forall \beta < \alpha (C \cap (\beta, \alpha) \neq \emptyset)}_{\alpha \text{ is a limit of ordinals from } C} \right\}$. Prove C' is a club subset of κ and $C' \subseteq C$.

(b) Show $D \stackrel{df}{=} \left\{ \gamma \in C \mid \forall \xi < \gamma (\eta(\xi) < \gamma) \right\}$ is club in κ for any $\eta : \kappa \rightarrow \kappa$.